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ABSTRACT

In this paper, two models of individual labour supply are discussed. The first one is the by now classical Hausman-type model with convex piece-wise linear budget constraints, in which both random preferences and optimization errors are incorporated by means of normally distributed random variables. Estimated coefficients are plausible but the model has the shortcoming that unemployment for males is not captured and that the simulated hours distribution misses the spikes in the sample distribution of working hours. Therefore, an alternative model is introduced which explicitly takes into account demand side restrictions on working hours. The difference with the standard model is the replacement of the optimisation error by the assumption that each individual can choose from a finite set of wage hours packages and either picks the job offer yielding highest utility or decides not to work. It turns out that this model captures the sample distribution of working hours very well, for males as well as females. Wage and income elasticities according to the two models are similar and in line with other recent findings in the Netherlands. Dead weight loss calculations for the second model which explicitly take the hours restrictions into account, imply that the dead weight loss is much smaller than as calculated with the standard model.

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1. Introduction

Due to the pioneering work of Jerry Hausman, the treatment of piece-wise linear convex budget constraints in the analysis of labour supply is now a rather standard practice. See, for example, Hausman (1980, 1981), Blomquist (1983) and Moffitt (1986). In Section 2 of this paper we replicate this type of analysis on data for the Netherlands. Both the labour supply of women and of men is studied for a cross-section of Dutch households drawn in 1985. Some simulations based on the estimation results are performed to calculate elasticities and dead weight losses.

Although the standard model comes out with plausible coefficients and the results are well in line with earlier findings in the Netherlands, a simple simulation reveals that various features of the data are not reproduced. In particular, the model overpredicts participation by men, whereas for women the participation of various education groups is badly tracked by the model. A comparison of the distribution of hours worked in the sample with the hours distribution generated by the model makes clear that the simulated distribution is far too smooth. In particular, the model misses spikes at fourty hours a week for males and at 40, 20, and 32 hours a week for females.

All this suggests that, at least in the Netherlands, the assumption underlying the model that observed hours mainly reflect the outcome of unrestricted choices by individuals is incorrect. In Section 3, we introduce a simple reduced form model of the demand side of the labour market, in which employers offer wage hours packages and each individual can choose from a limited number of such offers. Out of these jobs, the one yielding the highest utility level is picked, or the individual decides not to work. It is also possible that someone is unemployed because he or she has received zero offers.

It turns out that this adjustment of the standard model makes a dramatic difference in terms of the explanation of male participation and the fit of the generated hours distribution to the observed distribution in the sample. Interestingly, in the calculation of elasticities we obtain results that are of the same order of magnitude as in the standard model. The results of the extended model suggest that for females the distribution of desired hours is situated to the left of the distribution of hours offered by the employers. This mismatch between both distributions is a possible explanation of the low participation rate of women in the Netherlands in comparison with most other developed countries. If women could all work their desired number of hours, the participation rate of married females would rise from about 40 % to about 80 %, although most of them would like to work 16 hours a week or less. Dead weight loss calculations for the extended model which explicitly take the hours restrictions into account, suggest that the welfare loss is much smaller than as calculated with the standard model.

All these results should be interpreted with caution. Although the model in Section 3 in some respects certainly performs better than the model in Section 2, it does not yield a perfect description of labour supply behaviour in the Netherlands. Test results show that misspecification is still present. In Section 4, some possible future extensions and improvements of the model are suggested. The main contribution of the present paper is perhaps not a reliable conclusion about 'the true labour supply elasticities' or a guideline for tax reforms. Much more, it is another warning against the temptation to stick to one particular model, without carefully investigating whether this model is able to explain certain features of the data to a sufficient extent.

2. Common model

Starting point of the analysis is a modified version of the model introduced by Hausman (1981):

$$h^*_{ij} = \beta w_{ij} + \delta N_{ij} + X'_j\alpha + \varepsilon_j \qquad (2.1)$$

$$\begin{aligned} h^*_j &= 0 && \text{if } h^*_{1j} < 0 \\ &= h^*_{ij} && \text{if } H_{i-1,j} \le h^*_{ij} \le H_{ij}, \quad i=1,\dots s \qquad (2.2) \\ &= H_{ij} && \text{if } h^*_{ij} > H_{ij} \text{ and } h^*_{i+1,j} < H_{ij}, \; i=1,\dots s-1 \end{aligned}$$

$$
\begin{aligned}
&= T && \text{if } h^*_{sj} > H_{sj} \\
h_j &= 0 && \text{if } h^*_j + \nu_j < 0 \text{ or } h^*_j = 0 \\
&= h^*_j + \nu_j && \text{if } h^*_j + \nu_j \geq 0 \text{ and } h^*_j > 0
\end{aligned} \tag{2.3}
$$

where

h^*_{ij} = utility maximizing number of working hours for individual j on the line containing budget segment i

w_{ij} = virtual hourly wage rate of individual j on budget segment i

N_{ij} = virtual non-labour income of individual j for budget segment i

X_j = vector of exogenous demographic variables of individual j

ε_j = random variable representing stochastic preferences

h^*_j = preferred number of working hours

H_{ij} = kink-points of the budget constraint ($H_{0j}=0$, $H_{sj}=T$)

T = total time available

h_j = observed number of working hours of individual j

ν_j = random variable representing measurement or optimization errors

s = the number of budget segments

β, δ, α: parameters.

The main difference between this model and the one in the Hausman (1981) paper is that random preferences (ε_j) are included in the constant term rather than in the income coefficient β. The error terms $(\varepsilon_j, \nu_j)'$ are assumed to be drawn from the bivariate normal distribution with mean $(0,0)'$ and covariance matrix

$$\begin{bmatrix} \sigma^2_\varepsilon & 0 \\ 0 & \sigma^2_\nu \end{bmatrix}.$$

The corresponding direct utility function and expenditure function are given by

$$U(h,c) = -\delta \, \frac{h-\delta c-X'\alpha-\varepsilon}{\beta-\delta h} - \log(\beta-\delta h) \tag{2.4}$$

and

$$e(w,u) = \{-e^{-\delta w-u}+\beta-\beta\delta w-\delta(X'\alpha+\varepsilon)\}/\delta^2 \tag{2.5}$$

respectively, where c in (2.4) denotes consumption and u in (2.5) denotes the utility level. The direct utility function is defined and quasi-concave on the set $\{(h,c);\ \beta-\delta h>0\}$, which contains the set $\{(h,c);\ h\geq 0\}$ if $\beta>0$ and $\delta<0$. In this case the model is also coherent in the sense that (2.1), (2.2) and (2.3) yield exactly one solution h_j for all ϵ_j and ν_j (cf Gourieroux et al. (1980) and Van Soest et al. (1988)).

Data and Estimation results

The data we used stem from a labour mobility survey carried out in the Netherlands in 1985 under auspices of the Organisation of Strategic Labour Market Research (OSA). The sample contains information on 849 families consisting of at least husband and wife. Some sample statistics are mentioned in Table 2.1. The sample contains 315 families in which both partners are employed, in 486 families only the husband is employed and in 16 cases only the female works. In the remaining 32 families both partners are unemployed. After tax wage rates of employed individuals were not directly observed but constructed from hours worked and after tax labour income. Before tax wage rates were then calculated by using an approximation of the Dutch income tax system. Some simplifying assumptions were necessary because the data-set did not contain all the necessary information on deductables, health insurance premiums, etc.

Making use of the computed before tax wage rates of workers, a before tax wage equation allowing for the possibility of selection bias was estimated for males and females seperately. The explanatory variables used were log(age) (LAGEM and LAGEF for males and females respectively),

log(age) squared,

dummies referring to different education levels (EDM and EDF), and

an index variable referring to the sector of education (EDSECM and EDSECF: 2=technical or business, 1=semi-technical or semi-business, 0=neither technical nor business).

Estimation results are mentioned in Appendix A. Making use of actual before tax wage rates for workers and predicted before tax wage rates for unemployed individuals, for each person a convex piecewise linear approximation of the budget set was constructed. Again, simplifying assumptions due to lack of information were necessary. Minor non-linearities and non-convexities due to e.g. thresholds in deductables were ignored, as well as unemployment benefits; only the basic system of at most eleven income brackets was explicitly taken into account.

Table 2.1. Sample statistics

Variable	Mean	St. dev.	Minimum	Maximum	Number
LOGFS (logarithm of family size)	1.18	0.37	0.69	2.30	849
DCH<6 (dummy children younger than 6)	0.26	0.44	0	1	849
LAGEM (logarithm of age, male)	3.65	0.26	3.00	4.14	849
LAGEF (logarithm of age, female)	3.58	0.27	2.89	4.11	849
L2AGEM (LAGEM-squared)	13.38	1.87	8.97	17.17	849
L2AGEF (LAGEF-squared)	12.88	1.94	8.35	16.90	849
EDM (education level male)	2.78	1.08	1	5	849
EDF (education level female)	2.35	1.01	1	5	849
EDSECM (education sector male)	0.95	0.99	0	2	849
EDSECF (education sector female)	0.29	0.70	0	2	849
WRATM (after tax wage rate, male)*	15.97	5.80	6.87	59.47	801
WRATF (after tax wage rate, female)*	12.54	4.53	5.81	39.38	331
WRBTM (before tax wage rate, male)*	27.90	13.80	8.94	174.55	801
WRBTF (before tax wage rate, female)*	19.27	8.06	7.35	60.65	331
HM (working hours per week, male)*	42.07	6.70	4	71	801
HF (working hours per week, female)*	27.29	12.52	2	60	331

Note: *: working individuals only

The model was estimated by maximum likelihood using the algorithm of Berndt et al (1974). The calculation of bivariate probabilities was based upon a formula given by Abramowitz and Stegun (1970), p. 940. Estimation results are given in Table 2.2.

Table 2.2 Estimation results of the common model
(Standard errors in parentheses)

Parameter	Males		Females	
β (wage rate)	0.51	(0.17)	1.29	(0.44)
δ (unearned income)	-0.0055	(0.0035)	-0.0080	(0.0039)
α_0 (constant term)	-153	(104)	-489	(203)
α_1 (LOGFS)	0.18	(1.77)	-31.8	(7.0)
α_2 (DCH<6)	-0.94	(1.33)	-21.4	(5.1)
α_3 (LAGE)	108	(58)	325	(120)
α_4 (L2AGE)	-15.3	(7.9)	-50.0	(17.3)
σ_ε (random pref.)	12.93	(0.49)	19.19	(5.36)
σ_ν	3.22	(1.40)	12.77	(3.23)

The estimates for the wage rate coefficients are significantly different from zero and imply that labour supply is forward bending. The income effects have the expected negative sign and for females, the coefficient differs significantly from 0. Both the wage and the income effect are stronger for females than for males. The impact of family characteristics (family size and the presence of young children) is insignificant for males and strongly significant for females. The estimates imply that, ceteris paribus, labour supply is maximal at 34 and 26 years of age for males and females respectively. The estimated standard deviations of the random variables ε and ν are significantly different from zero. For males in particular, random preferences seem to be the most important source of random variation of observed hours worked.

Simulations and Computation of Dead Weight Loss

Table 2.3 provides sample means and a simulation of the actual 1985 situation in order to see to what extent the model is able to describe the data. In this table, means of hours worked and average participation probabilities are given for males and females divided into groups according to several different individual characteristics. Sample means of hours worked (zeroes included) in column 2 can be compared with simulated means (column 3) in which both sources of random variation are taken into account (i.e. for each individual 10 different random drawings of ε and ν are used). The results show for instance that for females, the model captures the differences in hours worked between people of different levels of education to some extent but not completely. This may be a consequence of the fact that education was not included as an explanatory variable in the vector X of individual characteristics but it may also be due to the restrictive way in which hours are allowed to depend on the wage rate. The differences in the average numbers of hours worked for different age levels and family sizes appear to be well captured. For males, there are hardly r y differences left to be explained.

The other columns of Table 2.3 refer to participation probabilities, i.e. the sample participation fraction (column 4), the simulated fraction if only random preferences are taken into account (column 5), and the fraction with both sources of random variation taken into account (column 6). The numbers in column 5 may be interpreted as probabilities of desired participation. Actual participation observed may differ from this because of demand side restrictions, measurement errors, suboptimal behaviour, etc, all the error sources included in ν. For females, predicted participation is slightly larger than the observed participation in the sample. Again, the fact that differences in preferences between females of different education levels are not fully captured by the model becomes apparent. For males, the model hardly appears to explain any non-participation at all. Predicted participation exceeds 0.997 for all groups, whereas actual participation in the sample equals 0.944. This shortcoming of the model may e.g. be due to the fact that fixed costs of working are not incorporated or to the fact that demand side restrictions (involuntary unemployment) are not explicitly taken into account.

Table 2.4 shows the consequences of a 10 % increase of all after tax wage rates. The aggregate average number of hours worked would increase by 6.6 % for females and 1.2 % for males. In relative terms, the change is similar for groups with different education levels, age or family size. The relatively large increase of female working hours is partly explained by the rise in the average participation probability of 5.6 %. For males, the simulated degree of participation was already almost equal to one before the wage increase and thus hardly changes. A simulation of a 10 % decrease of after tax wage rates shows results that are almost symmetric to the results shown in Table 2.4: The aggregate average of hours worked falls by 6.3 % for females and 1.2 % for males. Income elasticities are computed in the same way. A 10 % rise of all unearned incomes (virtual incomes due to the tax system excluded) leads to a fall in average hours worked for males of only 0.1 %. For females, the fall is 2.3 %, mainly because the average participation probability falls by 2.1 %.

Whereas Table 2.3 contains information on average numbers of working hours, Figures 2.1 and 2.2 refer to the actual and simulated distributions of working hours. Frequencies of zero hours of work are not included in the figures (these frequencies can be obtained from Table 2.3). The figures present the actual sample distributions for all males and all females respectively and present two different simulated hours distributions: the distribution with all sources of random variation taken into account, i.e. the distribution of h_j-s given by (2.3), and the distribution of the h_j^*-s given by (2.2). The latter can be interpreted as the distribution of desired working hours, since the errors included in ν_j, which reflect several sources of deviations from optimal behaviour, are not taken into account. The gap between desired hours frequencies and simulated actual frequencies for females reflects involuntary unemployment, as far as explained by this model: A negative realisation of ν_j implies that someone who would like to work few hours a week does not actually work. The probability of involuntary unemployment thus is a strongly decreasing function of desired labour supply.

The figures show that the model is not able to explain the spikes at 40 hours of work for males and females and at 20 hours of work for females. This is a usual shortcoming of labour supply models in the Netherlands which do not take into account any forms of hours restrictions and it motivates the explicit modelling of such restrictions. In Section 3 such a model will be discussed.

The results of dead weight loss (DWL) calculations based on the estimation results are mentioned in Table 2.5. DWL was calculated ten times for each individual, with different random drawings of ε. The random variation through ν was not taken into account. For a given individual and given ε, h^* and corresponding consumption c^* were determined using (2.1) and (2.2). Then (2.4) and (2.5) were used to compute the lump sum tax which generates the same utility level as the actual tax system. The table contains the average DWL for groups with different characteristics, in absolute terms (Dfl per week, col. 6) as well as in relative terms, i.e. as a fraction of the tax revenues in the original system (col. 7). The average DWL is approximately 32.6 % and 30.3 % for males and females respectively of the average amount of taxes paid according to the actual system. As was to be expected, DWL is highest for people with large labour supply, since their marginal tax rate is largest. For the same reason, one would expect DWL for males to exceed DWL for females. This is true in absolute terms but not in relative terms, because the female's own wage elasticity is larger than the male's. The effect on hours worked of changing the actual tax system into a system of lump sum taxes is illustrated in columns 2 and 3 of the table, which contain average predicted numbers of working hours for the actual and the lump sum system. The larger the differences between the numbers in these columns, the larger are the dead weight losses. DWL was also calculated ignoring random preference variation, i.e. with both ϵ and ν set equal to 0. The results were quite similar to those mentioned in Table 2.5.

Conclusion

The estimation results of the standard model are satisfactory in the sense that all parameter estimates have the expected signs. Moreover, estimated wage and income elasticities are largely in accordance with what we would expect intuitively. On the other hand however, simulation of the actual situation and in particular the figures comparing sample distributions with simulated distributions reveal important shortcomings of the model: It does not capture the spikes in the male and female hours distribution and it cannot explain non-participation among males. In order to test the specification of the model formally, it was also estimated using information on employed individuals only (with a conditional likelihood function). For males as well as females, some of the resulting parameter estimates were quite different from the original ones and standard errors of the estimates were smaller instead of larger. Therefore, the formal Hausman test statistics (see Hausman, 1978) were not computed, but the misspecification intuitively became more apparent. These estimation results are mentioned in Appendix B. The figures in this appendix show that these estimates still imply that the spikes in the male and female hours distribution are not explained.

Several extensions of the model can be suggested to overcome the shortcomings. In our opinion, the explicit incorporation of existing demand side or institutional restrictions on hours worked seems a very important one, at least in the Netherlands. This approach is taken in the remainder of this paper.

Table 2.3. Simulation of the actual 1985 situation

Males		1	2	3	4	5	6
All males:		849	39.70	39.40	0.943	0.999	0.999
Educ. level:	1	134	37.69	39.31	0.903	0.999	0.999
	2	167	37.05	38.94	0.910	0.999	0.998
	3	342	40.54	39.05	0.965	0.999	0.999
	≥4	206	41.75	40.42	0.961	1.000	1.000
Age:	< 30	154	39.60	39.09	0.955	0.999	0.999
	30-39	300	40.83	40.40	0.960	1.000	1.000
	40-49	225	39.96	39.94	0.924	0.999	0.999
	≥ 50	170	37.43	37.23	0.929	0.999	0.998
Family size:	2	263	39.98	39.18	0.958	1.000	1.000
	3	140	38.71	39.30	0.929	0.999	0.998
	4	282	38.85	39.75	0.929	0.999	0.999
	≥ 5	164	41.52	39.25	0.957	0.999	0.999
Females							
All females:		849	10.64	9.94	0.390	0.487	0.412
Educ. level:	1	217	6.73	7.75	0.290	0.403	0.332
	2	223	6.94	8.77	0.278	0.452	0.374
	3	322	13.26	11.30	0.450	0.522	0.452
	≥ 4	87	20.18	13.38	0.701	0.660	0.557
Age:	< 30	226	20.35	18.25	0.593	0.709	0.637
	30-39	296	7.84	8.34	0.334	0.445	0.372
	40-49	212	7.66	5.71	0.330	0.388	0.306
	≥ 50	115	4.25	5.53	0.243	0.344	0.266
Family size:	2	263	23.32	21.30	0.711	0.821	0.741
	3	140	6.71	7.64	0.329	0.486	0.390
	4	282	5.65	5.24	0.277	0.371	0.289
	≥ 5	164	2.25	1.76	0.122	0.153	0.113

Explanation:

column 1: number in the sample
2: hours worked, sample mean
3: hours worked, simulated
4: participation, sample
5: desired participation, simulated
6: actual participation, simulated

Table 2.4 Simulation of a 10 % increase of after tax wage rates

Males		1	2	3	4	5	6
All males:		849	39.70	39.89	0.943	0.999	0.999
Educ. level:	1	134	37.69	39.77	0.903	0.999	0.999
	2	167	37.05	39.40	0.910	0.999	0.999
	3	342	40.54	39.54	0.965	0.999	0.999
	≥ 4	206	41.75	40.94	0.961	1.000	1.000
Age:	< 30	154	39.60	39.53	0.955	0.999	0.999
	30-39	300	40.83	40.88	0.960	1.000	1.000
	40-49	225	39.96	40.44	0.924	0.999	0.999
	≥ 50	170	37.43	37.74	0.929	0.999	0.999
Family size:	2	263	39.98	39.64	0.958	1.000	1.000
	3	140	38.71	39.78	0.929	0.999	0.999
	4	282	38.85	40.25	0.929	0.999	0.999
	5	164	41.52	39.76	0.957	0.999	0.999
Females							
All females:		849	10.64	10.59	0.390	0.513	0.434
Educ. level:	1	217	6.73	8.26	0.290	0.427	0.350
	2	223	6.94	9.36	0.278	0.478	0.393
	3	322	13.26	11.99	0.450	0.547	0.476
	≥ 4	87	20.18	14.35	0.701	0.691	0.595
Age:	< 30	226	20.35	19.09	0.593	0.729	0.654
	30-39	296	7.84	8.97	0.334	0.470	0.396
	40-49	212	7.66	6.29	0.330	0.419	0.333
	≥ 50	115	4.25	5.98	0.243	0.372	0.290
Family size:	2	263	23.32	22.23	0.711	0.837	0.762
	3	140	6.71	8.36	0.329	0.524	0.424
	4	282	5.65	5.81	0.277	0.403	0.314
	≥ 5	164	2.25	2.04	0.122	0.174	0.126

Explanation:

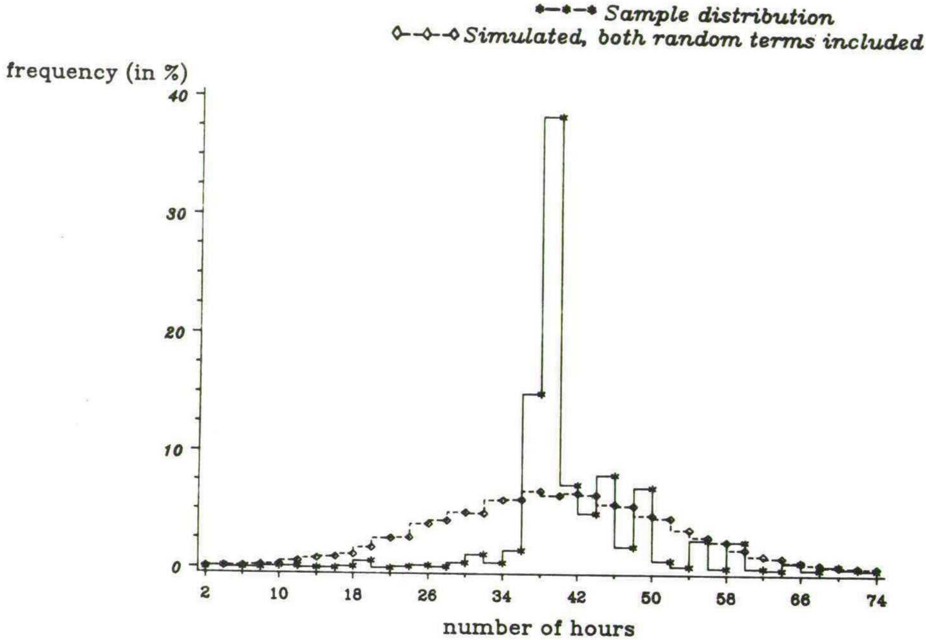
column 1: number in the sample
2: hours worked, sample mean
3: hours worked, simulated
4: participation, sample
5: desired participation, simulated
6: actual participation, simulated

Table 2.5 Dead weight loss calculations

Males	1	2	3	4	5	6	7
All males:	849	39.39	45.88	191	253	62	0.33
Educ. level:1	134	39.26	43.35	111	132	21	0.19
2	167	38.90	43.31	120	141	21	0.18
3	342	39.03	44.90	169	213	44	0.26
≥ 4	206	40.45	51.25	336	489	153	0.45
Age: < 30	154	39.08	42.91	110	124	14	0.12
30-39	300	40.34	47.16	203	268	65	0.32
40-49	225	39.89	47.96	245	343	98	0.40
≥ 50	170	37.32	43.58	171	225	54	0.32
Family size:2	263	39.20	44.40	155	187	32	0.21
3	140	39.31	45.54	181	236	55	0.30
4	282	39.78	46.93	210	280	70	0.33
≥ 5	164	39.08	46.72	224	328	104	0.46
Females							
All females:	849	9.33	11.11	16	21	5	0.30
Educ. level:1	217	7.24	8.22	9	10	2	0.18
2	223	7.92	9.19	11	13	2	0.22
3	322	10.73	12.80	20	26	6	0.30
≥ 4	87	12.92	17.01	36	53	16	0.45
Age: < 30	226	17.77	20.74	32	38	7	0.21
30-39	296	7.65	9.56	16	24	7	0.44
40-49	212	5.20	6.18	6	8	2	0.31
≥ 50	115	4.67	5.26	4	5	1	0.24
Family size:2	263	20.75	24.63	41	52	11	0.26
3	140	6.75	8.17	10	14	5	0.53
4	282	4.54	5.46	5	8	2	0.46
≥ 5	164	1.44	1.66	1	1	0	0.34

Explanation:

column 1: number in the sample
2: hours simulated, actual tax system
3: hours simulated, lump sum taxes
4: taxes, actual system (Dfl per week)
5: taxes, lump sum (Dfl per week)
6: dead weight loss (Dfl per week)
7: (dead weight loss)/(taxes actual system)



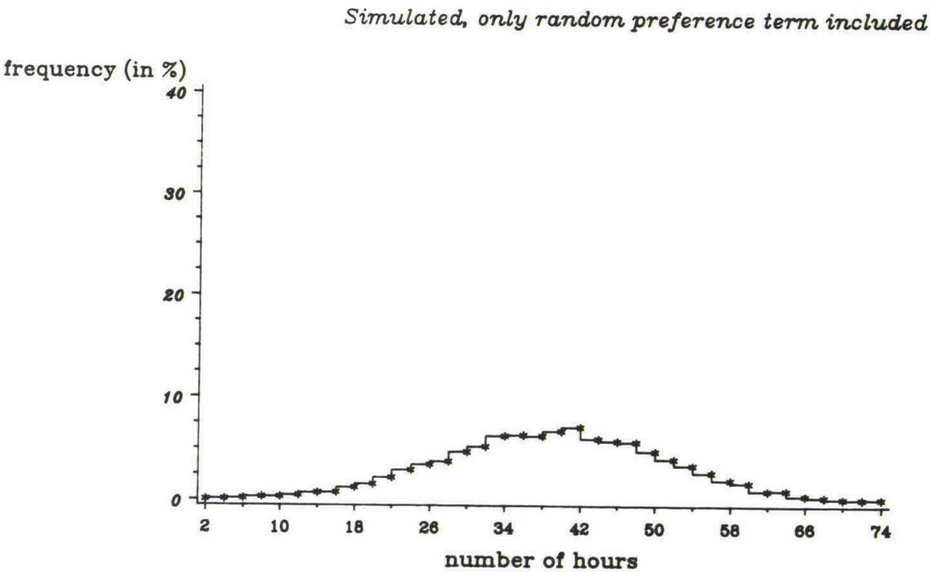
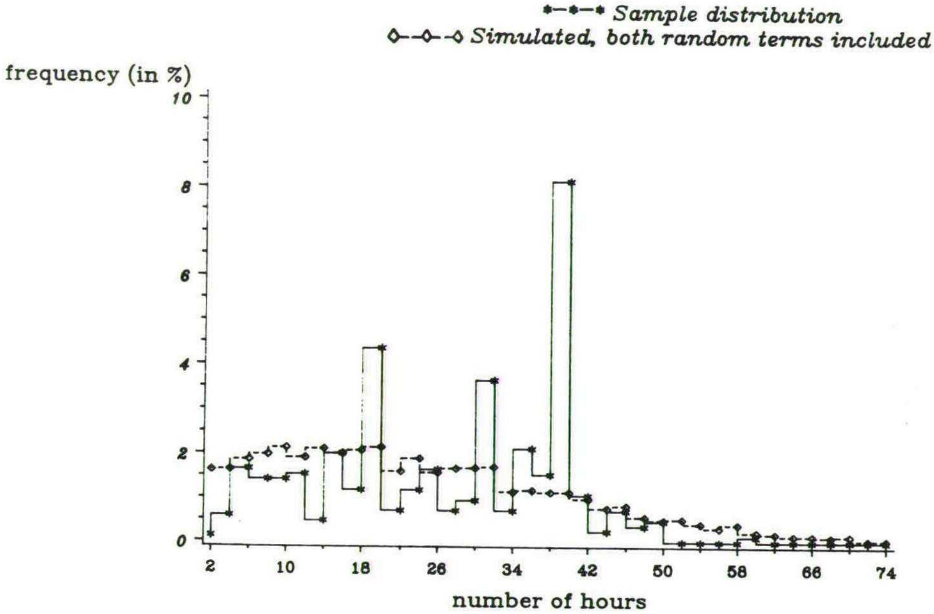


Figure 2.1 Distribution of working hours per week
Males, common model



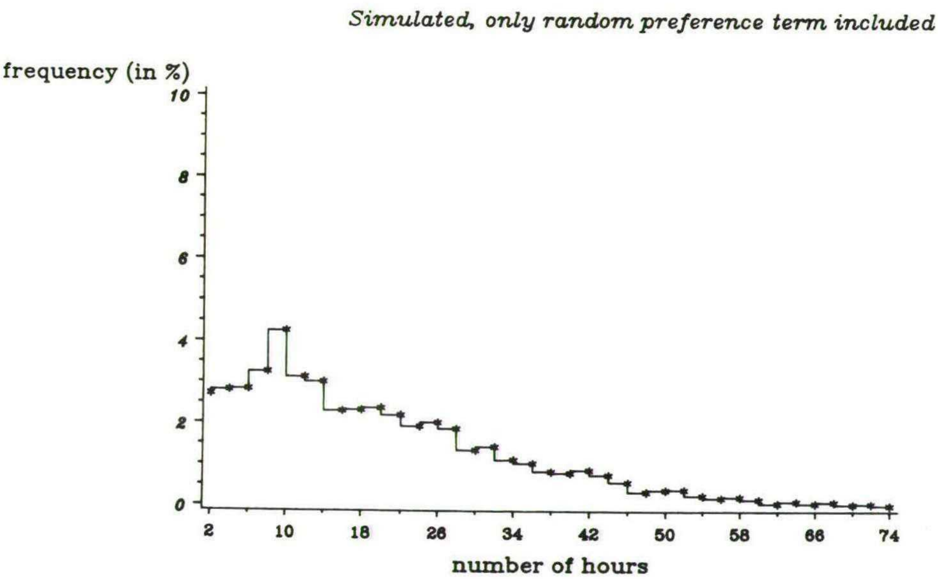


Figure 2.2 Distribution of working hours per week
Females, common model

3. A Model with Demand Side Restrictions

In Section 2, a labour supply model was introduced that took into account tax laws in describing the budget constraint. In this section we present a model which also explicitly captures demand side restrictions by modelling the limited availability of jobs with different, distinct, numbers of hours. Examples of labour supply models in which hours restrictions and involuntary unemployment are taken into account are Moffitt (1982), Ham (1982) and Blundell et al. (1987). The model studied here is largely based on a paper by Dickens and Lundberg (1985).

The Model

The starting point is the common model, described in equations (2.1)-(2.2). The error term ν, representing among other things deviations from preferred numbers of hours due to demand side restrictions, is omitted. Instead, we model demand side restrictions by means of distributional assumptions about job offers. Another difference with the common model is that the number of hours of work is no longer considered to be a continuous variable, but a discrete one. In this section we consider numbers of hours at 4-hour intervals, i.e. 0, 4, 8, etc. In what follows, we use the index ℓ to denote such hours points h. For example, $\ell=0$ corresponds with h=0, $\ell=1$ with h=4, $\ell=2$ with h=8 and so on. For clarity of presentation we omit both the index j, denoting the j-th individual and the index i, denoting budget segment i (compare equation (2.1)).

We assume that employers offer jobs with fixed numbers of hours. Workers face the market distribution of these employment opportunities. Furthermore, it is assumed that the market distribution of job offers is the same for all workers, such that the probability that one job offer involves $h_\ell(\neq 0)$ working hours is:

$$\Pr[\text{one job offer } h=h_\ell] = p_\ell , \qquad \ell=1,\ldots,m. \tag{3.1}$$

Here m is the number of different values of working hours $h_\ell>0$. An individual receives N job offers which are not necessarily different, i.e. he may for example receive N job offers all requiring 40 hours per week.

The probability of this event (conditional on N) is p_{10}^{N} (ℓ=10 corresponds to h=40). The number of job offers an individual receives, N, is assumed to be a drawing from a binomial distribution $B(N_{max},P_{of})$. The maximum number of job offers N_{max} is fixed at 10.

In this context the labour supply decision becomes a discrete choice between N job offers, drawn from the market distribution of offers, or not working:

$$h = h_k \quad \text{iff} \quad U(h_k,c_k) \geq U(h_\ell,c_\ell) \tag{3.2}$$

for all ℓ in the range of received job offers and for ℓ=0. Here c_ℓ is the consumption level corresponding to h_ℓ. If there are no demand restrictions, i.e. an individual can choose any number of hours, then (3.2) holds for all ℓ. In general, the individual still maximizes utility, but on a <u>subset</u> of possible numbers of hours only. For a non-worker this subset may contain only one element, zero hours, because N is assumed to be a random variable of which zero is one of the possible outcomes. In this way the model distinguishes between voluntary and involuntary unemployment. The main idea is that an individual is only observed to work h_k(>0) hours if he received at least one job offer h_k and if he prefers this job offer to all different job offers he received and to unemployment. The individual is unemployed if he prefers zero hours of work to all job offers he received or if he received zero offers.

The likelihood contribution of a given observation is a function of:

- parameters of the utility function (β, δ, α-s, σ_ε)
- probabilities of job offers with different numbers of hours (p_k-s)
- the parameter determining the number of job offers an individual receives (P_{of}).

In this model there are three sources of randomness, namely:

- ε, representing stochastic preferences,
- N, the number of job offers, and
- the offered numbers of hours.

Let $R_k(\varepsilon,N)$ be the conditional probability of observing $h=h_k$, for given ε and N (k=0,...,m). It is straightforward to determine $R_k(\varepsilon,N)$ from (3.2) because if preferences (i.e. ε) are known, it is easy to check for each

$\ell \neq k$ whether $U(h_k, c_k)$ exceeds $U(h_\ell, c_\ell)$ or not. Since the taste parameter ε is not observed, the likelihood of observing $h=h_k$ hours given N can be written as the mean of $R_k(\varepsilon, N)$:

$$L(h=h_k | N) = \int_{-\infty}^{\infty} f(\varepsilon)\, R_k(\varepsilon, N)\, d\varepsilon \ , \qquad (3.3)$$

where f is the density function of ε ($\sim N(0,\sigma_\varepsilon^2)$).
For random N the likelihood function (3.3) is given by:

$$L(h=h_k) = \sum_{N=0}^{N_{max}} L(h=h_k | N)\, p(N) \qquad (3.4)$$

where p is the probability function corresponding to $B(N_{max}, P_{of})$.
Since $L(h=h_k | N=0) = 0$ if $h_k \neq 0$, for workers equation (3.4) turns into

$$L(h=h_k) = \sum_{N=1}^{N_{max}} L(h=h_k | N)\, p(N) \qquad (3.5)$$

Since $L(h=0| N=0) = 1$, equation (3.4) can be written for non-workers as

$$L(h=0) = p(0) + \sum_{N=1}^{N_{max}} L(h=0| N)\, p(N) \qquad (3.6)$$

Thus unemployment can either be due to the fact that no job offers are received at all or to the fact that all job offers received are less attractive than not working. For more details about the model and the likelihood function, see Tummers and Woittiez (1988). The main difference with the common model is the fact that the error term ν is replaced by the job offer mechanism. Thus, an alternative explanation is given for differences between actual and desired labour supply: Instead of assuming that these deviations are random drawings from a normal distribution, flexibility is added in the sense that correlation of deviations from desired behaviour with the desired number of working hours is allowed for. As in the common model, the distinction between desired and actual labour supply hinges strongly on identifying assumptions, since no information on desired behaviour is used. In the extended model, the main identifying assumption is that the job offer distribution does not depend on individual characteristics such as age, education, etc. Therefore, conclusions about desired labour supply should be interpreted with caution; eventually, desired hours are only introduced as a tool to create a model which yields a reasonable description of the distribution of actual working hours.

Estimation Results

Table 3.1 presents estimation results of the model described above, which is referred to as the extended model. It was estimated for males and females seperately with the data set described in Section 2. As said before, the number of hours in this model is assumed to be a discrete variable. To each of the points with hours strictly greater than 0 corresponds a probability that this number of hours is offered. These probabilities could be estimated freely, but to reduce the number of parameters to be estimated we have set the probabilities of various points equal to each other. We have for example assumed that jobs involving 4, 8, 12, or 16 hours per week are offered with the same probability. For the exact distributional assumptions of job offers see the upper panel of Table 3.1. The maximum number of hours offered is set equal to the sample maximum of hours worked, i.e. 72 for males (m=18) and 60 for females (m=15).

The number of job offers an individual receives is a random drawing from the binomial distribution $B(10,P_{of})$. The estimated value of P_{of} is 1 (upper bound) for males and 0.395 for females. This implies that a man always receives 10 offers, whereas a woman only receives about 4 job offers on average. It follows from the numbers in Table 3.1 that according to this model most job offers involve 40 or more hours per week. The estimates for $p_{12}=\ldots=p_m$ seem rather large, for both men and women, and imply that almost everyone has the opportunity to work 48 hours or more. These large numbers however do not necessarily imply that many people actually work so many hours, since preferences are such that these offers will rarely be accepted (see below).

Table 3.1 **Estimation results of the extended model**

(Standard errors in parentheses)

		Males		Females	
job offers[1)]					
$p_1=\ldots=p_4$	(4,8,12,16)	0.0009	(0.0005)	0.012	(0.0039)
p_5	(20)	0.002	(0.001)	0.030	(0.001)
$p_6=p_7$	(24,28)	0.001	(0.0004)	0.015	(0.005)
p_8	(32)	0.003	(0.001)	0.050	(0.017)
p_9	(36)	0.006	(0.001)	0.036	(0.013)
p_{10}	(40)	0.297	(0.132)	0.302	(0.083)
p_{11}	(44)	0.309	(0.062)	0.130	(0.042)
$p_{12}=\ldots=p_m$	(48,...,4m)[2)]	0.054	(---)[2)]	0.094	(--)[2)]
P_{of}		1.0	(---)[3)]	0.395	(0.143)
preferences					
β (wage rate)		0.405	(0.242)	0.768	(0.243)
δ (unearned income)		-0.0007	(0.003)	-0.0041	(0.0013)
α_0(constant)		-259.	(109.)	-172.1	(85.5)
α_1(LOGFS)		-0.141	(1.733)	-14.1	(3.1)
α_2(DCH<6)		-0.899	(1.287)	-9.03	(2.22)
α_3(LAGE)		167.	(61.)	126.0	(49.9)
α_4(L2AGE)		-23.7	(8.4)	-19.8	(7.2)
σ_ε(random pref.)		11.91	(0.98)	7.71	(1.96)

Notes:

1) the number(s) of working hours to which the probabilities correspond are given in parentheses.

2) m=18 for males and m=15 for females; $p_{12}=\ldots=p_m$ is determined by the other probabilities because probabilities add up to one. Therefore, no standard error is computed.

3) since the estimate is at its upper bound, no standard error is computed.

The lower panel of Table 3.1 contains estimated parameter values of the utility function. It is striking that for females all estimated values are smaller in absolute value than the corresponding ones in the common model, but they are still significant. The presence of children strongly reduces (ceteris paribus) the female's desired number of working hours, whereas for males, family characteristics only play a minor role. The age profiles of preferred hours do not differ much from those found in Section 2. Hours rise with age until about 32 years for men and 25 years for women. There is a remarkable difference between the estimates for σ_ε for females in the two models. Apparently, part of the variation in actual hours worked which the common model explained by different preferences, is ascribed to differences in hours restrictions by the extended model.

Simulations

Table 3.2 provides simulation results of the extended model. This table is comparable with Table 2.3, on the understanding that an extra column is added with simulated desired working hours. The main differences between the simulations with the common model (Table 2.3) and the extended model (Table 3.2) are:

- Differences in hours worked between people of different levels of education are better explained by the extended model (but not completely).
- Simulated desired participation for females is much larger in the extended model than in the common model, implying that according to the extended model females are strongly restricted in their choice: There are not enough jobs with a limited number of working hours (See also Figure 3.2 below).
- Unlike the common model, the extended model is capable of explaining the 6% non-participation of males. Since simulated preferred participation is close to one, non-participation is explained by demand side restrictions: All males receive 10 job offers, but some of them only receive offers involving an unattractive (large) number of working hours and thus they prefer not to work.

Table 3.2 Simulation of the actual 1985 situation

Males		1	2	3	4	5	6	7
All males:		849	39.70	37.01	40.25	0.943	0.999	0.944
Educ. level:	1	134	37.69	36.06	39.37	0.903	0.998	0.931
	2	167	37.05	36.41	39.70	0.910	0.999	0.936
	3	342	40.54	37.11	40.33	0.965	0.999	0.945
	≥ 4	206	41.75	37.95	41.13	0.961	0.999	0.958
Age:	< 30	154	39.60	36.54	39.80	0.955	0.999	0.937
	30-39	300	40.83	38.55	41.51	0.960	0.999	0.960
	40-49	225	39.96	37.53	40.74	0.924	0.999	0.952
	≥ 50	170	37.43	34.05	37.78	0.929	0.998	0.912
Family size:	2	263	39.98	36.36	39.67	0.958	0.999	0.935
	3	140	38.71	36.69	39.96	0.929	0.999	0.940
	4	282	38.85	37.62	40.78	0.929	0.999	0.952
	≥ 5	164	41.52	37.29	40.50	0.957	0.999	0.948
Females								
All females:		849	10.64	12.45	10.89	0.390	0.825	0.396
Educ. level:	1	217	6.73	10.18	8.01	0.290	0.753	0.313
	2	223	6.94	11.75	9.69	0.278	0.822	0.366
	3	322	13.26	13.50	12.35	0.450	0.850	0.435
	≥ 4	87	20.18	16.05	15.72	0.701	0.925	0.534
Age:	< 30	226	20.35	18.74	20.11	0.593	0.928	0.636
	30-39	296	7.84	11.34	9.17	0.334	0.816	0.354
	40-49	212	7.66	9.60	6.50	0.330	0.794	0.283
	≥ 50	115	4.25	8.24	5.27	0.243	0.707	0.238
Family size:	2	63	23.32	20.72	22.82	0.711	0.976	0.717
	3	140	6.71	11.59	8.59	0.329	0.878	0.356
	4	282	5.65	9.34	5.87	0.277	0.805	0.269
	≥ 5	164	2.25	5.30	2.35	0.122	0.575	0.133

Explanation:

column 1: number in the sample
2: actual hours worked, sample mean
3: preferred hours, simulated
4: actual hours worked, simulated
5: participation, sample
6: desired participation, simulated
7: actual participation, simulated

Table 3.3 Simulation of a 10% increase of after tax wage rates

Males	1	2	3	4	5	6	7
All males:	849	39.70	37.48	40.64	0.943	0.999	0.949
Educ. level: 1	134	37.69	36.49	39.75	0.903	0.999	0.936
2	167	37.05	36.84	40.08	0.910	0.999	0.941
3	342	40.54	37.57	40.72	0.965	0.999	0.950
≥ 4	206	41.75	38.47	41.55	0.961	1.000	0.963
Age: < 30	154	39.60	36.94	40.15	0.955	0.999	0.942
30-39	300	40.83	39.01	41.88	0.960	1.000	0.964
40-49	225	39.96	38.01	41.13	0.924	0.999	0.957
≥ 50	170	37.43	34.54	38.25	0.929	0.998	0.920
Family size: 2	263	39.98	36.79	40.05	0.958	0.999	0.941
3	140	38.71	37.16	40.36	0.929	0.999	0.946
4	282	38.85	38.10	41.17	0.929	0.999	0.957
≥ 5	164	41.52	37.78	40.90	0.957	0.999	0.953
Females							
All females:	849	10.64	13.18	11.75	0.390	0.849	0.421
Educ. level: 1	217	6.73	10.81	8.71	0.290	0.779	0.335
2	223	6.94	12.46	10.52	0.278	0.846	0.392
3	322	13.26	14.25	13.24	0.450	0.873	0.460
≥ 4	87	20.18	16.97	16.97	0.701	0.942	0.568
Age: < 30	226	20.35	19.45	21.03	0.593	0.939	0.658
30-39	296	7.84	12.11	10.10	0.334	0.842	0.383
40-49	212	7.66	10.35	7.33	0.330	0.824	0.311
≥ 50	115	4.25	8.84	5.92	0.243	0.734	0.260
Family size: 2	263	23.32	21.45	23.88	0.711	0.980	0.740
3	140	6.71	12.35	9.61	0.329	0.898	0.387
4	282	5.65	10.10	6.71	0.277	0.835	0.297
≥ 5	164	2.25	5.93	2.79	0.122	0.620	0.152

Explanation:

column 1: number in the sample
2: actual hours worked, sample mean
3: preferred hours, simulated
4: actual hours worked, simulated
5: participation, sample
6: desired participation, simulated
7: actual participation, simulated

Table 3.4 Simulation of an increase of the number of part-time job offers

<u>Males</u>	1	2	3	4	5	6	7
All males:	849	39.70	37.01	38.80	0.943	0.999	0.968
Educ. level: 1	134	37.69	36.06	37.89	0.903	0.998	0.959
2	167	37.05	36.41	38.23	0.910	0.999	0.963
3	342	40.54	37.11	38.89	0.965	0.999	0.968
≥ 4	206	41.75	37.95	39.71	0.961	0.999	0.976
Age: < 30	154	39.60	36.54	38.34	0.955	0.999	0.963
30-39	300	40.83	38.55	40.18	0.960	0.999	0.977
40-49	225	39.96	37.53	39.31	0.924	0.999	0.973
≥ 50	170	37.43	34.05	36.12	0.929	0.998	0.948
Family size: 2	263	39.98	36.36	38.18	0.958	0.999	0.962
3	140	38.71	36.69	38.50	0.929	0.999	0.965
4	282	38.85	37.62	39.37	0.929	0.999	0.972
≥ 5	164	41.52	37.29	39.07	0.957	0.999	0.970
<u>Females</u>							
All females:	849	10.64	12.45	11.54	0.390	0.825	0.486
Educ. level: 1	217	6.73	10.18	8.90	0.290	0.753	0.399
2	223	6.94	11.75	10.58	0.278	0.822	0.461
3	322	13.26	13.50	12.80	0.450	0.850	0.523
≥ 4	87	20.18	16.05	15.94	0.701	0.925	0.631
Age: < 30	226	20.35	18.74	19.39	0.593	0.928	0.711
30-39	296	7.84	11.34	10.14	0.334	0.816	0.451
40-49	212	7.66	9.60	7.85	0.330	0.794	0.382
≥ 50	115	4.25	8.24	6.54	0.243	0.707	0.326
Family size: 2	263	23.32	20.72	21.87	0.711	0.976	0.798
3	140	6.71	11.59	10.08	0.329	0.878	0.475
4	282	5.65	9.34	7.40	0.277	0.805	0.372
≥ 5	164	2.25	5.30	3.35	0.122	0.575	0.190

<u>Explanation:</u>

column 1: number in the sample
2: actual hours worked, sample mean
3: preferred hours, simulated
4: actual hours worked, simulated
5: participation, sample
6: desired participation, simulated
7: actual participation, simulated

The consequences of a 10% increase in all after tax wage rates are presented in Table 3.3. It shows a small increase (1.0%) in hours worked for men, and a larger increase (7.9%) for women. These results are similar to those in the common model. It is interesting to see that for females the elasticity of the actual number of hours worked with respect to the own wage rate is larger than the corresponding elasticity of desired hours (5.9%). This may be explained by the fact that the choice set is discrete: Some females will not react at all if their wage rate increases, but for others the discrete 'jump' may exceed the rise in preferred hours. Apparently, the second effect slightly dominates the first.

Income elasticities are obtained in the same way. If unearned incomes rise with 10%, male labour supply hardly changes. For females, actual hours and participation decrease by 2.3% and 2.0% respectively. Preferred hours and participation fall by 2.0% and 1.0%.

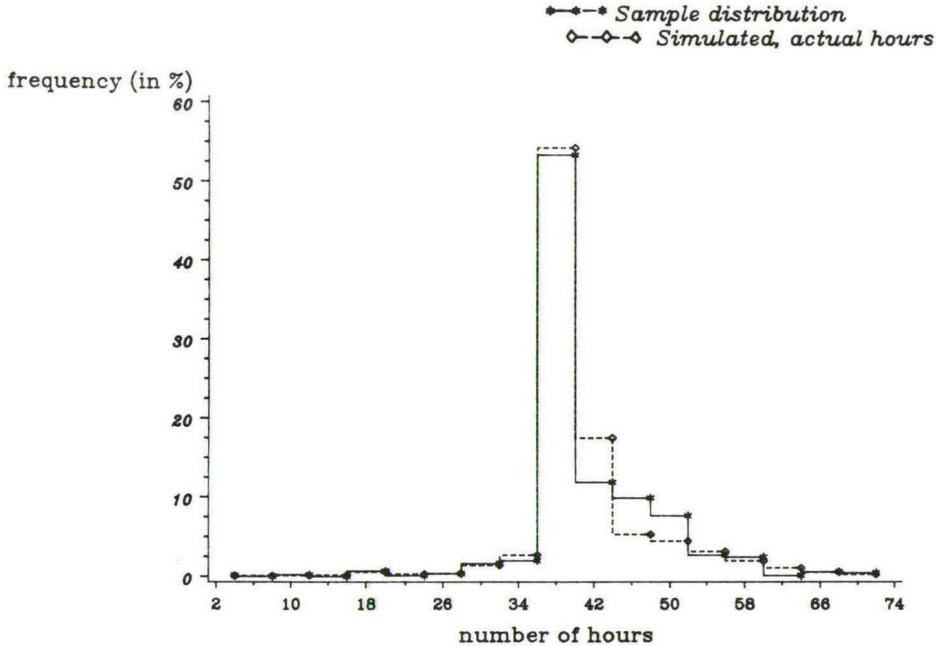
In a final simulation the consequences are studied of relaxing hours restrictions in the sense that more part-time jobs are offered, i.e. jobs involving 20 hours per week. According to the estimation results, the probability that at least one 20 hours a week job is offered is 2.3% for males and 11.4% for females. Table 3.4 shows what happens if the value of p_5 (the probability that one offer involves 20 hours) is increased in such a way that the probability of receiving at least one offer of 20 hours becomes 50%. (For males, this implies p_5=0.067, and as a consequence $p_{12}=..=p_{18}$ is reduced to 0.045; for females it implies p_5=0.168 and $p_{12}=..=p_{15}$=0.0592). Because restrictions are relaxed, actual numbers move towards preferred ones. Thus many of those who prefer to work part-time but either did not work or worked full time because there was no part-time job opportunity, will now be able to find a 20 hours job. Non-participation and full-time work will fall in favor of part-time work. For males, the fall in the number of full-time workers dominates and average working hours decrease for all age and education categories. 12.5% of all males will choose to work 20 hours a week. For females, the fall in the number of people who do not work dominates and the average number of working hours rises for all groups. Non-participation falls by 15% and 23.5% of all women will work 20 hours.

The parameter estimates in the lower panel of Table 3.1 were used to simulate the distribution of preferred hours, given in Figures 3.1 and 3.2

for men and women respectively. These figures can be compared with the simulated hours distributions without measurement or optimisation errors, presented in Figures 2.1 and 2.2. The two distributions are very similar apart from differences due to the different desired participation probabilities which we already discussed (See Table 3.2).

Combining the demand side of the model (i.e. the offers distribution) and the supply side (preferred hours) one obtains the distribution of actual working hours such as it is simulated with the extended model. Comparing the sample distribution of actual hours with the simulated distribution shows that th- extended model predicts an hours distribution much more in line with the data than the common model. This improvement must be attributed to the different distributional assumptions. By assuming that both random variables in the common model are normally distributed, one forces an hours distribution which is too smooth. In the extended model this is no longer the case.

Figures 3.1 and 3.2 only display information about working individuals. Let us now focus on the 61% non-participating females and about 5.6% non-participating males. The extended model predicts a degree of participation of respectively 60.5% and 5.5%. Table 3.5 yields information on how these numbers come about. In the first column the simulated actual hours distribution is given, and in the second the simulated preferred distribution. Only 17.5% of the females prefer not working to any other number of hours. The remaining 43% of predicted non-participants is due to restrictions on the demand side. A large number of women, 54%, prefer to work between 4 and 16 hours per week. But jobs requiring such low number of hours are rarely offered: The last column contains the probability that the choice set contains the number of hours h_{ℓ}, i.e. $1-(1-P_{of}p_{\ell})^{N_{max}}$ $(\ell=1,\dots,m)$. This column again shows that for almost everyone the choice set contains the opportunity of working full-time or more, but that many do not have the option of working part-time. Intuitively, the fact that firms are reluctant to offer jobs with few hours a week might well be explained by the existence of fixed employer costs for each separate employee. Again however, it should be stressed that this interpretation hinges strongly on the identifying assumptions. An alternative explanation for the lack of part-time jobs may



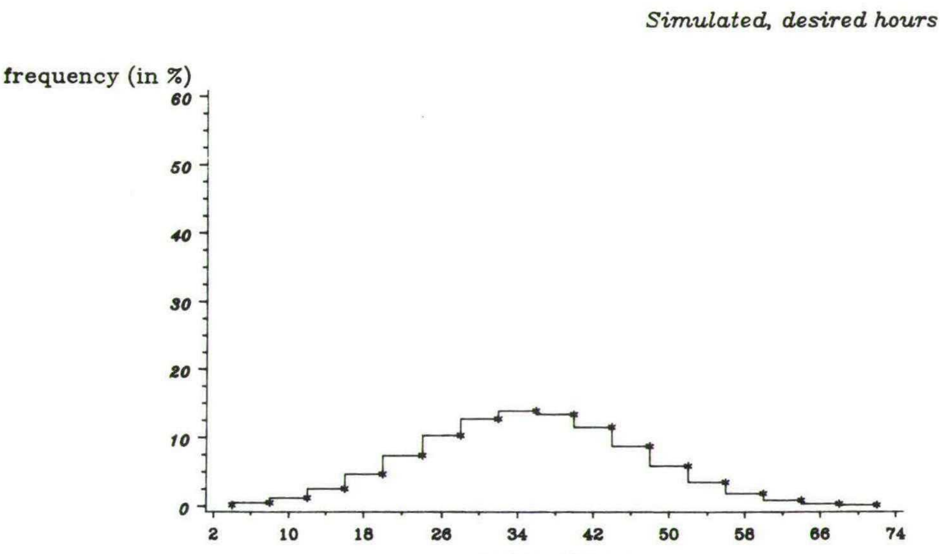
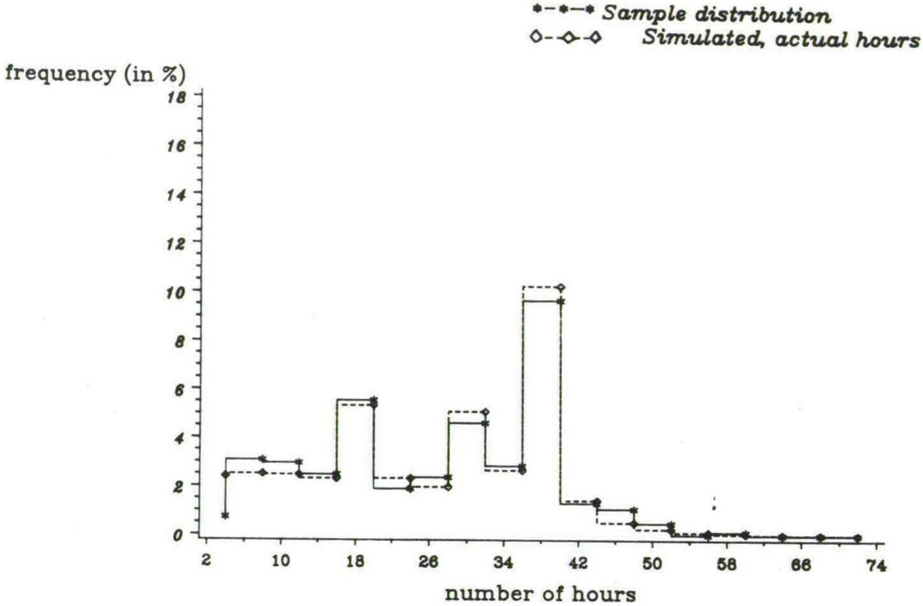


Figure 3.1 Distribution of working hours per week
Males, extended model



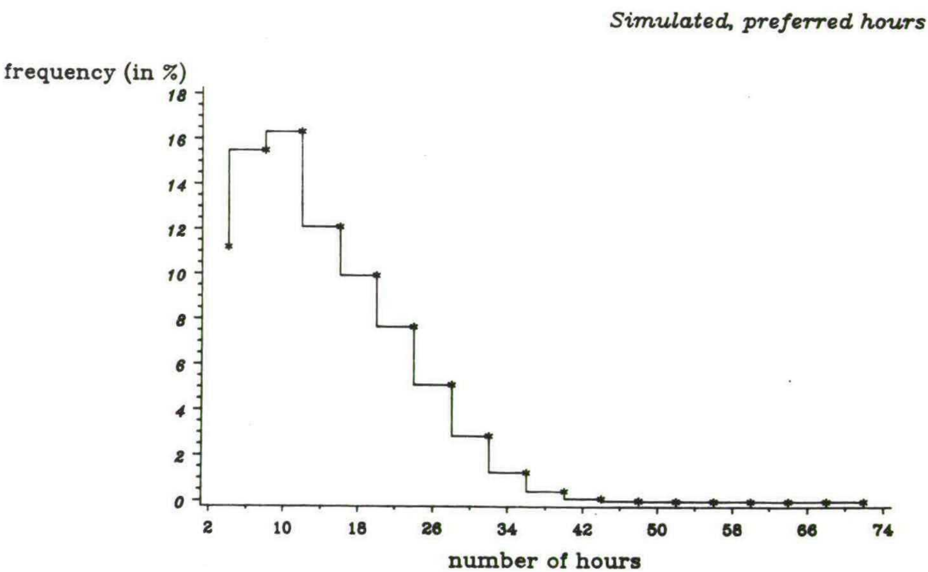


Figure 3.2 Distribution of working hours per week
Females, extended model

for instance be the existence of fixed costs at the supply side of the labour market, such as costs of child care, etc, which are not taken into account in either the common or the extended model. These fixed costs make it unattractive for an individual to work only a few hours. In the extended model the rare occurrence of people working only a few hours is explained by demand side restrictions.

Table 3.5 Simulated, actual preferred and offered hours distributions
(Probabilities x 100)

	males			females		
hours(h_ℓ)	actual	pref.	offered	actual	pref.	offered
0	5.58	0.11	0.9	60.44	17.45	-
4	0.07	0.20	0.9	2.40	11.19	4.6
8	0.10	0.54	0.9	2.50	15.50	4.6
12	0.13	1.24	0.9	2.47	16.32	4.6
16	0.17	2.59	0.9	2.31	12.11	4.6
20	0.59	4.70	2.4	5.33	9.94	11.3
24	0.32	7.38	1.1	2.32	7.67	5.8
28	0.38	10.30	1.1	1.96	5.12	5.8
32	1.42	12.71	3.4	5.07	2.86	18.1
36	2.71	13.85	5.8	2.64	1.26	13.3
40	54.25	13.35	97.0	10.24	0.44	72.3
44	17.52	11.49	97.5	1.41	0.12	42.0
48	5.29	8.73	42.5	0.50	0.02	34.1
52	4.46	5.88	42.5	0.23	0.00	34.1
56	3.16	3.53	42.5	0.10	0.00	34.1
60	1.96	1.89	42.5	0.04	0.00	34.1
64	1.09	0.90	42.5	0.02	0.00	34.1
68	0.55	0.38	42.5	0.01	0.00	34.1
72	0.25	0.22	42.5	0.00	0.00	34.1

Dead weight loss calculations

When dead weight losses are calculated, restrictions on working hours are usually ignored. Since the extended model explicitly takes these restrictions into account, it seems natural to incorporate the hours restrictions also in the calculation of DWL.

In order to understand the exact meaning of DWL in the context of hours restrictions, let us first consider the case in which individual preferences are known with certainty (i.e. ε is given) and the set of job offers is given, i.e. the individual chooses the best point in the finite set $\{(h^0,c^0),(h^1,c^1),\ldots,(h^N,c^N)\}$, where $h^0=0$ and c^i denotes consumption corresponding to h^i according to the actual tax system. The two figures below show what DWL may look like. Figure 3.3 refers to a person with a choice set containing six points. The optimal choice is (h^2,c^2), as can be see from the form of the indifference curve corresponding to utility level $U(h^2,c^2)$. Taxes paid will thus equal T. (Note that in the first tax bracket no taxes are paid, so the before tax wage rate equals w_1, the after tax (marginal) wage rate corresponding to the first bracket.) If the tax system is replaced by a lump sum tax T, the individual will choose to work more (i.e. h^3 or h^4) hours and his utility level will increase. Let c^{2j} be the consumption level defined implicitly by

$$U(h^2,c^2) = U(h^j,c^{2j}) \qquad (j=2,\ldots,N) \tag{3.7}$$

The dead weight loss is defined as the amount by which the lump sum must increase such that the optimal utility level - with hours restrictions taken into account - according to the lump sum system equals $U(h^2,c^2)$, so

$$DWL = \max_{j=2,\ldots,N} \{w_1 h^j + c^0 - c^{2j}\} - T \tag{3.8}$$

In Figure 3.3, the maximum is attained for j=3.

If in the case referred to by Figure 3.3 (h^3,c^3) and (h^4,c^4) were omitted from the choice set, then DWL would be zero. In that case, the individual would still work h^2 hours if the tax system was replaced by a lump sum T.

Figure 3.4 refers to someone with a choice set consisting of two different points only. Confronted with the actual tax system, this person chooses not to work and pays no taxes, but still the dead weight loss is non-zero. Note that in this example DWL would be zero if hours worked could be chosen freely, since in that case the individual would work h^* hours and changing the tax system would have no effect. This example thus shows that the presence of hours restrictions does not necessarily imply that DWL is smaller than it would have been if hours are chosen freely, as might be thought intuitively.

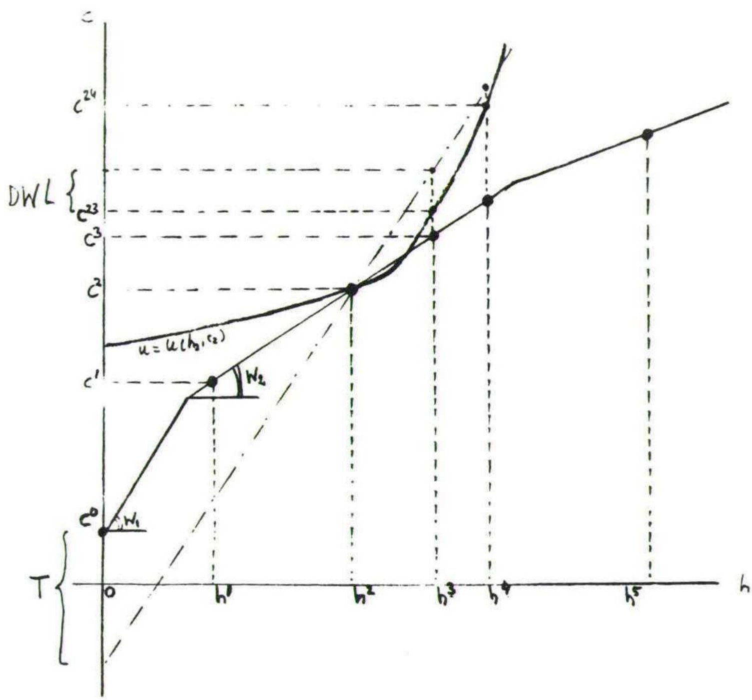


Figure 3.3 Dead weight loss for a finite set of job offers

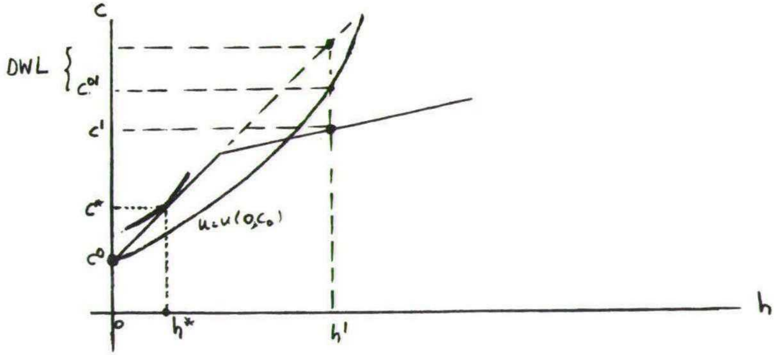


Figure 3.4 Dead weight loss in case of non-participation

It is straightforward to extend the examples given above to the 'general case' with preferences given by (2.4) with ε known and given job offers $\{(h^0,c^0),(h^1,c^1),\dots,(h^N,c^N)\}$: For $i,j \in \{0,\dots,N\}$, $i \leq j$, let c^{ij} be defined by

$$U(h^i,c^i) = U(h^j,c^{ij}) \tag{3.9}$$

An explicit expression for c^{ij} is easily derived from (2.4):

$$c^{ij} = \frac{1}{\delta^2}\,[\{\beta-\delta h^j\}\{U(h^i,c^i)+\log(\beta-\delta h^j)\} + \delta\{h^j-X'\alpha-\varepsilon\}] \tag{3.10}$$

Let (h^{i^*},c^{i^*}) be the utility maximizing choice. The dead weight loss is given by

$$\begin{aligned} DWL &= \underset{j=i^*,\dots,N}{\mathrm{Max}} \{w_1 h^j + c^0 - c^{i^*j}\} - \{w_1 h^{i^*} + c^0 - c^i\} = \\ &= \underset{j=i^*,\dots,N}{\mathrm{Max}} \{w_1(h^j-h^{i^*}) + c^i - c^{i^*j}\}. \end{aligned} \tag{3.11}$$

If preferences or the set of job offers are not fixed, we assume that the lump sum can be adjusted exactly to each possible realisation of the random preference term ε and to each set of job offers, and we are interested in the expectation of the dead weight loss. Let us first consider the case that ε is given but job offers are random. For $i,j \in \{0,1,\dots,m\}$, $i \leq j$, let

$$DWL_{ij} = \{w_1(h_j-h_i) + c_i - c_{ij}\}, \tag{3.12}$$

where c_i is consumption corresponding to h_i and c_{ij} is defined in the same way as c^{ij} in (3.9). DWL_{ij} is the realisation of DWL if in case of the actual tax system the optimal choice is (h_i,c_i) and in case of lump sum taxation the optimal choice is (h_j,c_j). Since $DWL_{ii} = 0$ the expected dead weight loss (conditional on ε) is given by

$$E\{DWL|\varepsilon\} = \sum_{i=0}^{m} \sum_{j=i+1}^{m} \Pr[DWL=DWL_{ij}]\, DWL_{ij} \tag{3.13}$$

The probabilities $\Pr[DWL=DWL_{ij}]$ can be calculated as follows: Let

$$\Omega_i = \{k\in\{0,\ldots,m\};\ U(h_k,c_k)\leq U(h_i,c_i)\} \quad (i=0,\ldots,m) \tag{3.14}$$

and

$$\Delta_{ij}=\{k\in\{0,\ldots,m\};\ k<i \text{ or } DWL_{ik}\leq DWL_{ij}\} \quad (i,j\in\{0,\ldots,m\}, i<j) \tag{3.15}$$

Thus, Ω_i corresponds to the offers which are not preferred to (h_i,c_i), and Δ_{ij} corresponds to the offers which yield a dead weight loss that does not exceed DWL_{ij}, conditional on the fact that (h_i,c_i) is the optimal choice. Therefore, $DWL=DWL_{ij}$ if and only if for each job offer (h_k,c_k) we have $k \in \Omega_i\cap \Delta_{ij}$ and (h_i,c_i) as well as (h_j,c_j) are offered. For $A \subset \{0,1,\ldots,m\}$ let

$$\begin{aligned} q(A) &= (1-P_{of}) + \sum_{k\in A\setminus\{0\}} P_{of}\, p_k && \text{if } 0 \in A \\ &= 0 && \text{if } 0 \notin A \end{aligned} \tag{3.16}$$

We avoid conditioning on the number of job offers N by interpreting the N_{max} - N "missed job offers" as offers of zero hours (which are of no importance, since zero hours of work can be chosen anyhow). Thus if $0\in A$ $q(A)$ can be interpreted as the probability that one job offer belongs to A. For each $i,j \in \{0,1,\ldots,m\}$ with $i<j$ we can now write

$$\begin{aligned} \Pr[DWL=DWL_{ij}]=&\{q(\Omega_i\cap\Delta_{ij})\}^{N_{max}}-\{q(\Omega_i\cap\Delta_{ij}\setminus\{i\})\}^{N_{max}} \\ &-\{q(\Omega_i\cap\Delta_{ij}\setminus\{j\})\}^{N_{max}}+\{q(\Omega_i\cap\Delta_{ij}\setminus\{i,j\})\}^{N_{max}} \end{aligned} \tag{3.17}$$

This implies that $\Pr[DWL=DWL_{ij}]=0$ if $U(h_i,c_i)<U(h_0,c_0)$ (i.e. $0\notin\Omega_i$) or if $U(h_i,c_i)<U(h_j,c_j)$ (i.e. $j\notin\Omega_i$). The expected dead weight loss can now be found by substitution of (3.17) into (3.13).

Finally, the expectation has to be taken with respect to ε also:

$$E(DWL) = E_\varepsilon\{E\{DWL|\varepsilon\}\} = \int_{-\infty}^{\infty} f(\varepsilon)\ E\{DWL|\varepsilon\}\ d\varepsilon. \tag{3.17}$$

Since $E\{DWL|\varepsilon\}$ is a complicated function of ε, it is not possible to compute the expectation analytically. Therefore, for each individual 10 values ε_j ($j=1,\ldots,10$) are drawn randomly from a $N(0,\sigma_\varepsilon^2)$-distribution and $E\{DWL\}$ is approximated by $\frac{1}{10}\sum_{j=1}^{10} E\{DWL|\varepsilon_j\}$.

Mean dead weight losses are given in Table 3.6, which can be compared to Table 2.5. According to the extended model, DWL appears to be much smaller than according to the common model, for both males and females. On average, DWL in the extended model is 10.7% of taxes paid for males and 15.4% for females. In the common model these figures were 32.6% and 30.3% respectively. Since wage rate elasticities are approximately the same in the two models, the differences must be due to the hours restrictions. The average change in hours worked (the difference between columns 3 and 2) for males is much smaller in the extended model than in the common model, but for females this is not the case. Moreover, the change from the actual tax system to lump sum taxation in the model with hours restrictions has a positive impact on participation (the effect described in the example illustrated in Figure 3.4), as can be seen from columns 8 and 9 in Table 3.6. Particularly for females this seems to play an important role: participation increases by 9.7%. However, the effect of this on total taxes paid is only small, since those who change from non-participation to participation choose to work relatively few hours a week. The extra tax revenues should mainly come from those who already paid a large amount and apparently for these people the hours restrictions play the largest role.

Table 3.6 Dead weight loss calculations

Males

		1	2	3	4	5	6	7	8	9
All males:		849	40.28	42.71	203	224	22	0.11	0.944	0.954
Educ. level:	1	134	39.17	40.82	116	124	8	0.07	0.922	0.933
	2	167	39.56	41.33	127	135	8	0.06	0.931	0.942
	3	342	40.51	42.75	184	200	17	0.09	0.947	0.956
	≥ 4	206	41.21	44.98	351	402	50	0.14	0.963	0.974
Age:	< 30	154	39.73	41.05	115	119	4	0.04	0.935	0.940
	30-39	300	41.66	44.30	218	241	23	0.11	0.961	0.971
	40-49	225	41.13	44.08	259	293	34	0.13	0.961	0.969
	≥ 50	170	37.20	39.58	181	199	18	0.10	0.898	0.917
Family size:	2	263	39.54	41.45	161	172	11	0.07	0.933	0.943
	3	140	39.85	42.15	189	208	19	0.10	0.935	0.944
	4	282	40.75	43.46	225	250	25	0.11	0.952	0.963
	≥ 5	164	41.03	43.90	243	278	35	0.15	0.955	0.965

Females

		1	2	3	4	5	6	7	8	9
All females:		849	10.84	12.63	23	27	4	0.15	0.395	0.433
Educ. level:	1	217	8.39	9.45	12	13	1	0.09	0.321	0.345
	2	223	9.23	10.82	16	18	2	0.13	0.354	0.391
	3	322	12.37	14.18	27	30	3	0.13	0.434	0.471
	≥ 4	87	15.43	19.46	57	71	14	0.25	0.533	0.615
age:	< 30	226	19.88	21.84	38	41	3	0.08	0.633	0.666
	30-39	296	9.20	11.30	26	31	6	0.22	0.353	0.399
	40-49	212	6.34	7.98	13	15	3	0.22	0.278	0.319
	≥ 50	115	5.59	6.51	8	9	1	0.13	0.247	0.270
Family size:	2	263	22.83	25.24	49	54	4	0.09	0.718	0.756
	3	140	8.72	10.86	20	25	5	0.23	0.360	0.411
	4	282	5.79	7.54	12	16	4	0.32	0.267	0.311
	≥ 5	164	2.11	2.67	3	4	1	0.25	0.125	0.142

Explanation:

column 1: number in the sample
2: hours simulated, actual tax system
3: hours simulated, lump sum taxes
4: taxes, actual system (Dfl per week)
5: taxes, lump sum (Dfl per week)
6: dead weight loss (Dfl per week)
7: (dead weight loss)/(taxes actual system)
8: simulated participation, actual tax system
9: simulated participation, lump sum taxes

4. Conclusions

In Sections 2 and 3, two models of individual labour supply are estimated, both of them based on the linear Hausman (1981) specification and accounting for a piece-wise linear budget constraint. Although some of the parameter estimates seem substantially different in the two models, calculated elasticities are quite similar. In a surve paper, Theeuwes (1988) discusses 8 other recent Dutch labour supply studies. He gives 8 wage rate elasticities for hours worked of women, ranging from 0.20 to 3.23 with a mean of 1.39. Compared to this, our elasticities of 0.65 and 0.79 are low but not out of line. For males, Theeuwes mentions 4 wage elasticities, ranging from -0.25 to 0.27 with a mean of 0.07. Our values (0.12 and 0.10) fit quite well in this range. The income elasticities that we find are also largely in accordance with previous Dutch results. Dead weight loss calculations yield quite different results for the two models. The DWL of 30% for females corresponds to values of 27% and 37%, which were obtained by Grift (1988) with Dutch 1983 data of married women. The substantially different DWL's found in Section 3 should perhaps not be too surprising, since the DWL definition hinges strongly on the structure of the model, i.e. on the demand side restrictions.

The introduction of the extended model is motivated by the fact that the common model yields a poor description of the sample distribution of working hours. The results, in particular Figures 3.1 and 3.2, unquestionably show that in this respect the model in Section 3 is a success. This however does not mean that it is free of misspecification. Several White tests for different subvectors of the parameter vector were performed and generally the null-hypothesis of no misspecification was rejected. Rejection was strongest for the parameters referring to demand side restrictions, intuitively suggesting that this is where most misspecification is located. On the other hand, for females, the hypothesis that P_{of} does not depend on LAGE and L2AGE is accepted on a 5 % level by a Lagrange multiplier test (a test statistic of 2.2 with critical value $\chi_{2;0.05}=6.0$).

A number of extensions and improvements of the model certainly deserve more attention in future research. The specification of preferences is convenient but possibly restrictive. For instance, preliminary analysis of a labour supply equation involving a quadratic wage term, suggests a significant improvement. This extension would correspond to the utility specification suggested by Hausman and Ruud (1984). Another interesting example is given by Heckman (1974), who starts with a specification of the indifference curves.

A second point relates to the treatment of the budget constraint for non-workers. For them, the budget constraint is based on predicted before tax wages. Random variation across individuals in before tax wages is ignored. This calls for simultaneous estimation of a wage equation and a labor supply equation. Examples of such models are Moffitt (1984), who introduces hours dependent wage rates resulting in an S-shaped budget curve, and Tummers and Woittiez (1988), who combine Moffitt's model with demand side restrictions. In the third place, the modelling of job offers in a static framework has the merit of simplicity, but a more natural approach would be to allow for consecutive job offers and the possibility that individuals move from one job to another. Somewhat in the same spirit it should be noted that budget constraints are not exogenous in a dynamic world. For instance, the level of unemployment benefits in the Netherlands often depends both on the duration of the preceding spell of employment and on the duration of the current spell of unemployment.

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Appendix A: wage equations

In estimating the labour supply models of Sections 2 and 3, unknown before-tax wage rates of non-workers were replaced by predicted wage rates. Predictions are based on the following estimation results of the log-wage equation. These were estimated following the Heckman procedure to take account of possible selectivity bias. For both males and females, the selectivity bias is not significant. The meaning of the exogenous variables is explained in Section 2 of the main text.

Table A.1 Wage equations

	Males		Females	
	Parameter	t-value[1)]	Parameter	t-value[1)]
Constant	-5.96	2.75	-12.71	4.49
DED2[2)]	0.063	1.54	0.091	1.58
DED3[2)]	0.144	3.93	0.160	3.24
DED4[2)]	0.393	10.10	0.468	8.11
LAGE	4.624	3.85	8.681	5.35
L2AGE	-0.600	3.58	-1.214	5.25
EDSEC[3)]	0.026	2.08	0.066	2.79
Lambda[4)]	1.379	1.39	-0.013	0.27
number of observ.	801		331	
R-squared	0.243		0.290	

Explanation:

1) t-values are not corrected for the possible selectivity bias.
2) DED2, DED3, DED4: dummy variables referring to the levels of education EDM (males) and EDF (females), ranging from 1 (lowest level) to 5 (highest level). DED2=1 if ED=2, DED3=1 if ED=3, DED4=1 if ED=4 or ED=5.
3) EDSEC: index variable referring to the sector of education; EDSEC=2: technical or business, EDSEC=1: semi-technical or semi-business, EDSEC=0: neither technical nor business.
4) Lambda: the inverse of Mill's ratio.

Appendix B: Estimation of the common model using information on workers only

The model described in Section 2 was also estimated using information on workers only, taking into account selectivity bias due to truncation by using conditional maximum likelihood. Note that this estimation procedure has the advantage that imputation of predicted wage rates for non-workers is avoided. Estimation results are mentioned in Table A.2. Some of the estimates are substantially different from those mentioned in Table 2.2, referring to the estimation with use of both workers and non-workers. These large differences strongly suggest that the common model is severely misspecified.

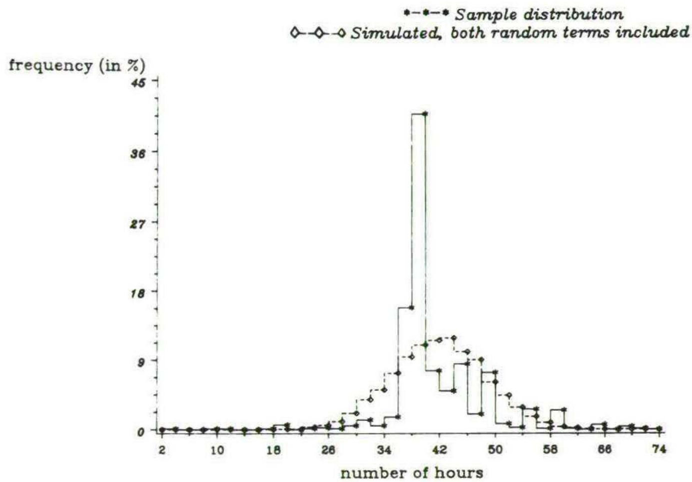
Table A.2 **Estimation results of the common model; workers only**
(Standard errors in parentheses)

Parameter	Males		Females	
β (wage rate)	0.0	$(---)^{*}$	0.91	(0.26)
δ (unearned income)	-0.0013	(0.0035)	-0.0061	(0.0028)
α_0 (constant term)	-154	(56)	25.1	(132)
α_1 (LOGFS)	0.32	(0.90)	-14.8	(2.6)
α_2 (DCH<6)	-0.17	(0.68)	-0.97	(2.4)
α_3 (LAGE)	109	(31)	21.0	(76.1)
α_4 (L2AGE)	-15.2	(4.3)	-5.1	(10.8)
σ_ε (random prefer.)	5.04	(11.6)	1.55	(4.98)
σ_ν	4.30	(13.4)	10.53	(0.64)

Note: *: the estimate is at its lower bound. This bound is imposed to avoid coherency problems.

Simulations based on these results can be compared with those mentioned in Tables 2.3 and 2.4. Predicted participation for males is almost equal to 1, as in Table 2.3. For females, the average simulated participation probability equals 0.957, which is quite out of line with both the actual sample participation and simulated participation in Table 2.3. The estimated average wage and income elasticities of actual working hours are both 0.0 for males. For females, they are 0.38 and -0.17 respectively.

Figures A.1 and A.2 are obtained in the same way as Figures 2.1 and 2.2 and can be used to compare the sample distribution of actual working hours with the simulated distribution based on the estimates of Table A.2. The figures show that excluding non-workers does not solve the problem that the spikes in the sample distribution of actual working hours are not explained.



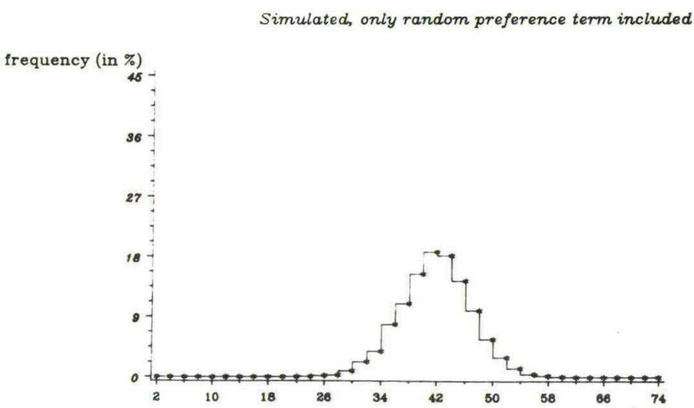
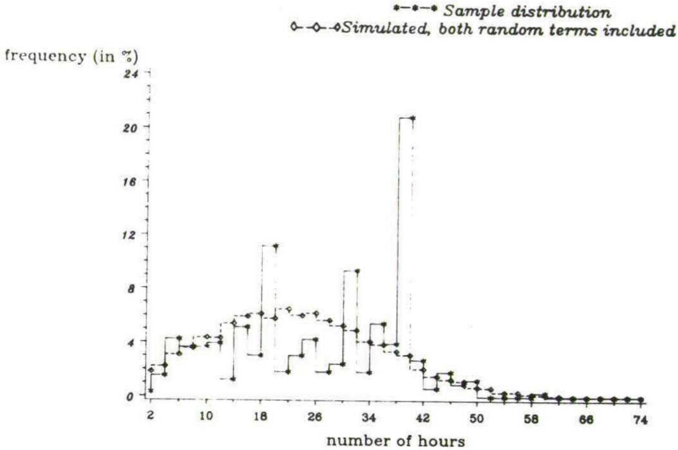


Figure A.1 Conditional distribution of working hours per week
Males, common model, workers only



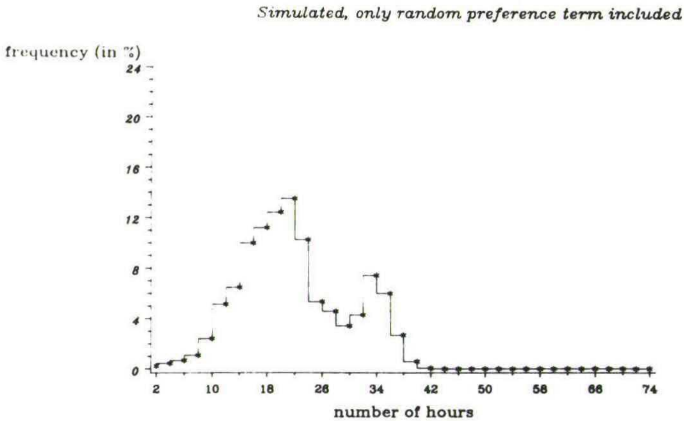


Fig: re A.2 Conditional distribution of . rking hours per w ek
Females, common model, workers only

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